• **5293:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

Let ABC be a triangle. Prove that

$$\sqrt[4]{\sin A\cos^2 B}+\sqrt[4]{\sin B\cos^2 C}+\sqrt[4]{\sin C\cos^2 A}\le 3\sqrt[8]{\frac{3}{64}}.$$

Comment: **Michael Brozinsky of Central Islip, NY and Kee-Wai Lau of Hong Kong China each** noticed that if $\triangle ABC$ has an obtuse angle, then the above inequality does not hold. This oversight can be corrected by restricting the statement of the problem to acute triangles.

Solution 2 by Arkady Alt, San Jose, CA

Since by AM-GM Inequality

$$\sqrt[4]{\frac{1}{2}\cdot\frac{\sin A}{\sqrt{3}}\cdot\cos^2 B}\ \le\ \frac{\frac{1}{2}+\frac{\sin A}{\sqrt{3}}+2\cos B}{4}\ \text{then}$$

$$\begin{aligned}\frac{1}{\sqrt[8]{12}}\sum_{cyc}\sqrt[4]{\sin A\cos^2 B} &= \sum_{cyc}\sqrt[4]{\frac{1}{2}\cdot\frac{\sin A}{\sqrt{3}}\cdot\cos^2 B}\le\sum_{cyc}\frac{\frac{1}{2}+\frac{\sin A}{\sqrt{3}}+2\cos B}{4}\\ &= \frac{3}{8}+\frac{1}{\sqrt{3}}\left(\sin A+\sin B+\sin C\right)+2\left(\cos A+\cos B+\cos C\right).\end{aligned}$$

Since $R\ge 2r$ (Euler Inequality) we have $\cos A+\cos B+\cos C=1+\frac{r}{R}\le\frac{3}{2}$.

Also, since $\sin x$ is concave down on $[0,\pi]$ then

$$\frac{\sin A+\sin B+\sin C}{3}\ \le\ \sin\frac{A+B+C}{3}=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\iff\sin A+\sin B+\sin C\le\frac{3\sqrt{3}}{2}.$$

Thus,

$$\frac{1}{\sqrt[8]{12}}\sum_{cyc}\sqrt[4]{\sin A\cos^2 B}\ \le\ \frac{1}{4}\left(\frac{3}{2}+\frac{1}{\sqrt{3}}\cdot\frac{3\sqrt{3}}{2}+2\cdot\frac{3}{2}\right)=\frac{3}{2}$$

$$\iff\ \sum_{cyc}\sqrt[4]{\sin A\cos^2 B}\le\frac{3}{2}\cdot\sqrt[8]{12}=3\sqrt[8]{\frac{3}{64}}.$$